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**MECHANICAL-GEOMETRICAL MODELLING IN NON-LINEAR
THEORY OF ELASTICITY**

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A new approach is proposed concerning the construction of constitutive relations of the elastic material under large deformations, which can be described as mechanical-geometrical. This approach significantly differs from the formal analytical method being based on polynomial approximation of the specific energy as the strain tensor's function. The approach is applied to the construction of the concrete model of an isotropic elastic media. The properties' analysis of the potential stress energy was carried out.

Keywords: non-linear elasticity, mechanical-geometrical model, specific strain energy.

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ASYMPTOTICS OF STATIONARY SOLUTIONS OF RAYLEIGH REACTION-DIFFUSION SYSTEM

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We consider Rayleigh reaction-diffusion system and study the bifurcational behavior of solutions, branching from zero due to the variation of control parameter. It is assumed that Dirichlet boundary conditions or mixed boundary conditions are set on the boundary of spatial domain. First terms of asymptotics are found explicitly, expressions for consecutive terms are derived. The qualitative dependence of solutions on boundary conditions is studied for one-dimensional (interval) and two-dimensional (rectangle) spatial domains. The evolution of solutions is studied numerically.

Keywords: Rayleigh system, Lyapunov-Schmidt reduction, monotonic instability, stationary solutions, asymptotic analysis, reaction-diffusion systems.

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MATHEMATICAL MODELING OF ROCKFALL USING THE CONTINUUM APPROACH

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The rockfall movement of the slope, which is associated with the horizontal section, was investigated. The theoretical investigation was performed using redistributable package OpenFOAM for solution of applied problems of hydro and aerodynamics. Continuum approach and kinetic theory of granular gas were used. Calculations were performed for various values of the slope angle to horizontal surface. Two-dimensional numerical calculations of distribution of rockfall mass were presented.

Keywords: mathematical modeling, rockfall, fluidization, continuum approach, kinetic theory of granular gas, OpenFOAM, twoPhaseEulerFoam.

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INTERPOLATIONAL AND COMBINATORIAL PROPERTIES OF KÖTHE POWER SPACES

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Interpolational (inner and outer) and combinatorial properties of Köthe power spaces are studied in this paper. This interpolational properties are expressed in terms of interpolation related to appropriate category of interpolation pairs, and combinatorial properties are expressed in terms of collection of coordinate subspaces of investigated Köthe spaces.

Keywords: Köthe spaces, power Köthe spaces, interpolation classes.

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