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ON EXACT SOLUTION OF MULTIDIMENSIONAL INTEGRAL EQUATION WITH HOMOGENEOUS KERNEL

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We study on the space $L_2(R^n)$ the multidimensional integral equation of the second kind, which kernel is homogeneous of degree $(-n)$, invariant with respect to the rotations group $SO(n)$ of the space R^n and satisfies to a certain summability condition. We suppose that the symbol of the equation is a non-degenerate that provides the equation is uniquely solvable for any free term. The purpose of this paper is to construct the solution of this equation. To do this, we use the special method based on the theory of spherical harmonics. Within this method, we perform a transition from a multidimensional integral equation to an infinite diagonal system of one-dimensional integral equations, whose kernels are homogeneous of degree (-1) . One-dimensional equations are considered on the space $L_2(R_+)$ and are uniquely solvable. By means of the Wiener theorem we construct the solutions of these equations and prove that they are coincide with the Fourier-Laplace coefficients of the sought solutions of the multidimensional equation. Moreover, the norms of operators, which determine these solutions, are bounded. The main result of our paper is Theorem 1, which establishes the formula of the solution of the original equation. This solution is constructed as a series by spherical harmonics, coefficients of which are the above-mentioned solutions of the one-dimensional equations. It is proved that this series converges on the space $L_2(R^n)$, and its sum is the solution of the original multidimensional equation.

Keywords: *integral equation, homogeneous kernel, symbol, spherical harmonics.*

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THE COMBINED MONTE-CARLO METHOD OF CALCULATION THE BARRIER OPTIONS FAIR PRICE

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The purpose of this work is to find the fair price of a barrier option under the Levy model with infinite intensity of jumps, by using the combined method, in which the probability of crossing the barrier by piecewise continuous trajectory of the stock price is calculated analytically through approximating the Levy process by Brownian bridges, and the expectation of the barrier option payment function over jump times is calculated numerically by using the Monte Carlo method. It have been shown that in case of the tempered stable process this method can be applied to the case of infinite variation, through approximation of the Brownian motion by small jumps. Special attention is given to the advantages of the models based on Levy processes in contrast with the classical Balck-Sholes model. It is noted that the required total number of random values for the Monte Carlo simulation can be reduced in contrast with the full simulation of the price process trajectory due to the lack of need to completely reproduce the trajectory of the stock price process. Numerical experiments to calculate the fair price of an barrier option in model under tampered stable process were conducted. The problem of the relationship between the normal approximation error and intensity of jumps was studied.

Keywords: *Levy process, option pricing, Monte Carlo method, Brownian bridge, model with jumps.*

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ON THE BASIS OF ITS OWN ELEMENTS OF THE SIXTH ORDER DIFFERENTIAL BUNDLE WITH TRIPLE CHARACTERISTICS

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Questions of basis functions on eigenfunctions of the linear differential operators go back to works of Fourier, Poisson, Cauchy, Liouville, Gilbert, Knezer and others. In the most common statement their decision for ordinary differential operators and bunches of operators was given in G. Birkhoff and J. D. Tamarkin's fundamental works. The defined notion of a regularity of boundary conditions of differential bunches which strictly assumed a differently of characteristic roots of the considered bunches and to which numerous followers in the spectral theory of bunches adhered was them. Our article falls into to studying of the theory of a differential bunch of the sixth order which is not falling into to a case of the regular in classical sense of spectral tasks with parameter. Special cases of such problems of orders 2, 3, 4, with the multiple roots of their secular equations, were considered in a number of works earlier. In this article, in case of a special differential bunch of the sixth order with two triple characteristics, the basis property of its characteristic elements is established. Difficulties, the bound to studying of tasks to the multiple characteristic roots are well-known (even in case of ordinary matrix operators). Importance of consideration of private types of such differential bunches which can specify a way to the common theory of bunches with the multiple characteristics is in this regard clear. We use the modified methods, the bound to construction and a research of a Green function of the corresponding task. The offered work serves one of chains in development of the theory of spectral tasks with the multiple characteristics, in case of ordinary differential bunches of operators.

Keywords: *Cauchy function, Green function, asymptotic, parameter, root functions.*

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APPLICATION OF SOFT AND PROBABILISTIC DECODERS FOR INTERCEPTED DATA RESTORING

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The possibility to restore noisy data received from channel interception with a soft and probabilistic decoding error-correcting codes. The following results were obtained. A general model of information-analytical channel monitoring system, which includes two legal members and the link connecting them, as well as one of the illegal member (observer). Data intercepting can be organized as through technical channels of information leakage, and via a physical connection to the existing transmission channel. The purpose of the legitimate participants - organization of secure transmission of confidential data from the sender to the recipient. The goal of the illegitimate member is unauthorized receipt of secure data, even if some of their distortions and with considerable delay in time. Formation of the interception is carried out channel to violate privacy only legitimate users of data, such as the channel does not involve feedback needed for violations, such as data integrity observer. The main part of the model is the model of illegitimate observer who organizes the interception of data channel from the legal channel. We describe the possible actions of the observer to obtain acceptable quality of data from illegitimate channel due to the high level of noise pollution due to the use of special decoder error-correcting codes, working outside half of the minimum distance. In connection with the building work model describes a common data transmission scheme, a description of some types of decoders. The results obtained on the basis of the following methodology: theoretical and probabilistic methods, simulation modelling and techniques of computational experiment. The results are applicable for designing the channels of confidential information transmission to assess the possibility of obtaining data by an observer on the borders of the controlled area.

Keywords: *monitoring channel, channel of information leakage, error-correction coding, probabilistic decoder, payroll decoder, soft decoder, information-analytical system.*

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2- AND 3-WAY ON A GRAPH-LATTICE AND COMBINATORIAL IDENTITIES

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Graph-lattice has vertices at the points of a plane with a non-negative integer coordinates. Each vertex has two oriented arcs: horizontal and vertical arcs to the neighboring vertices (right and top). The transition probability for each of the arc is equal to $1/2$. We considered the problem of random walks on the vertices of the graph with restrictions on the achievability. Valid for consideration are only 2(3)-paths. Under the 2(3)-path on the graph-lattice we mean a path of satisfying the next condition - maximum by embedding beginning sub-path consisting only of vertical arcs and all maximum by embedding internal sub-paths consisting only of vertical arcs have a length multiple of 2 (3). We obtain a formula for the number of paths connecting specified vertices of the graph-lattice, found the probability of getting from one vertex to another via 2(3)-paths. The consequence of the latter is the combinatorial identity, which contained elements of Pascal's triangle.

Keywords: directed graph, graph-lattice, random walk, transition probability, the attainability of the vertices, Pascal's triangle, combinatorial identity.

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THE COMPROMISE SOLUTIONS OF CLASSICAL COOPERATIVE GAME

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The subject of paper's investigation is the one-point solutions for transferable utility game that balance, in some sense, opposite principles of distribution of profit among the partners. First we describe known compromise solutions: the consensus value being the average of the Shapley value and the equal surplus division solution; the τ -value that balances the upper vector and the lower vector of a game; σ -values that equal the average of pairs of extreme points, one of which belongs to the imputation set whereas the other belongs to the dual imputation set. Axiomatic approach was used for the comparisons of solution concepts. Among the non-uniquely determined sets of characterizing axioms we chose the Shapley axioms and similar ones. The main result of the paper is an introducing the new solution for cooperative transferable utility game being an average of two most popular one-point solutions: the Shapley

value and the nucleolus. It is proved that for some games new solution, called NS-core, possesses more desirable properties than main one-point solutions and other compromise solutions. It is given the example of game in which NS-core does not coincide with the considered one-point solutions and is the most preferred outcome with respect to standard dominance relation. The application domains of the NS-core are the games that model economic, social and political situations where the Shapley value does not satisfy an individual rationality condition or it is not an element of a nonempty core, while the nucleolus gives a contrary-intuitive (paradoxical, tyrannous) sharing the joint profit.

Keywords: cooperative game, game value, compromise solution, consensus value, τ -value.

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