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INVESTIGATION OF WAVE PROCESSES IN AN INHOMOGENEOUS POROELASTIC LAYER

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A method for investigating wave fields in an inhomogeneous poroelastic layer of a transversely isotropic material is presented. The characteristics of the layer are considered to vary over thickness. To describe the poroelastic material, the model by M.A. Biot was chosen. The integral Fourier transform is applied to the basic equations written in terms of "displacement - pressure". The solutions of the transformed problem are constructed using the shooting method. The inversion of the Fourier transform performed numerically and using of the theory of residues. As auxiliary problems, the structure of the dispersion set of the considered problem is investigated and the displacement fields for an elastic layer with identical characteristics are constructed. A comparative analysis of the elastic and poroelastic problems is carried out. Estimates of the relative accuracy of the two methods of inversion of the transformation are obtained. The effect of the distribution law of the material characteristics of the layer on the dispersion set and displacement fields for the poroelastic layer is investigated.

Keywords: *poroelasticity, vibrations, layer, Biot's theory, theory of residues, numerical analysis.*

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TO CONTACT PROBLEMS FOR A CYLINDER

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The axially symmetric contact problem of the linear elasticity theory is investigated on the interaction between a rigid annular sleeve of finite length and an infinite hollow circular cylinder. The integral equation of the problem is derived by using a Fourier transformation. A new approximation for the kernel symbol of the integral equation of the problem is suggested to be effective for any cylinder wall thickness. On the basis of this approximation a singular asymptotic solution is constructed, the contact pressure and its integral characteristic are calculated for thin-walled cylinders. It is important that the parameters of the approximation are calculated with the help of the numerical Monte Carlo method. This method is especially fruitful for a big number of unknown parameters arising for thin-walled cylinders. Earlier for similar problems an approximation in the form of a sum of two different functions has been used giving only an approximate solution of the functional equation in the Wiener - Hopf method. A more simple approximation was used for a solid cylinder. The case of a finite hollow cylinder with end-walls subject to sliding support has been previously considered. The solution constructed can be useful for strength analysis of pipelines in contact with sleeves.

Keywords: *contact problem, elastic hollow cylinder.*

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THE SOLUTION OF THE PROBLEM OF FREE SPREADING OF THE WATER STREAM BEHIND THE OPEN WATER PIPE

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The equations of the movement of a rough water stream, two-dimensional in plan, in the plane of hodograf speed are recieved. This system of the equations of the movement of a stream is linear system of the equations in private derivatives unlike system of the equations in the physical plane of a current of a stream. The method of search of analytical decisions of system of the equations in the speed hodograf plane, which consists in division of variables in system and in search of a number of the replacements, leading the received equations to tabular (reference) is given in article. As a result, the decision of system of the equations is consolidated to the solution of the hypergeometric equation. Under various conditions of division of variables the analytical groups of decisions, allowing to put and solve analytically various boundary problems of a current of potential water streams, two-dimensional in plan, were received.

The boundary task of spreading of a potential water stream to widely taking away course behind a free-flow water throughput pipe of rectangular section is correctly set and solved in this work. The decision can be used for determination of parameters of a stream in the vicinity of an exit of a stream from a pipe to the wide smooth course as thus forces of resistance to a stream from a bottom of the course are small (they increase further downstream a stream with reduction of depth) and they can be neglected. The problem is solved at first in the plane of a hodograf speed and further stream parameters in physical area of a current of a stream are defined by integration of communications of transition to the physical plane.

The analytical formulas received in work allow to determine all range of parameters of a stream, thus convergence between model and natural or experimental parameters considerably exceeds convergence of parameters on earlier known methods.

Keywords: *water stream, rough stream, potential stream, water stream two-dimensional in the plan, hodograf of speed, the plane of hodograf speed, system of the differential equations in private derivatives, linear system of the equations, a method of division of variables, the hypergeometrical differential equation, a boundary task.*

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APPROXIMATE SOLUTION OF SINGULAR INTEGRAL EQUATIONS ON THE END OF UNLIMITED INTEGRATION OF CHEBYSHEV SERIES

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A computational scheme is constructed for the approximate solution of a singular integral equation of the first kind of an unbounded integration segment at the ends [– 1, 1]. *The solution of the equation is sought in the form of a series in Chebyshev polynomials of the first kind. The kernel and the right-hand side of the equation decompose into series using the Chebyshev polynomials of the first kind, whose coefficients are calculated approximately by Gaussian quadrature formulas. For the coefficients of the decomposition of Chebyshev polynomials of the second kind into series in Chebyshev polynomials of the first kind, exact values are found. The coefficients of the expansion of the unknown function, that is, the solution, are found from the solution of linear algebraic equations.*

To justify the computational scheme, methods of functional analysis and the theory of orthogonal polynomials are used. We introduce the space of Holder functions with the corresponding norms. In this space, we consider the given singular and corresponding approximate operators. Conditions for the existence of an inverse singular operator are given and the existence of an inverse approximate operator is proved. When the existence condition for the given functions having derivatives up to some order belonging to the Holder class is satisfied, the error in the computation is evaluated and the order of its tendency to zero is given.

Keywords: *singular integral, quadrature formula, Chebyshev series, expansion coefficients.*

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CONTACT PROBLEM FOR ELASTIC BLOCK TAKING INTO ACCOUNT FRICTION AND WEAR

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One of the most important factors varying exploitation time for friction units is wear. Thus, simulation for such process is vital problem of mechanical engineering. In literature one can find a lot of information on various wear models. Mathematical formulation of such models varies from simple empiric to complex equation, basing on physical definitions. Parameters and variables, used in such models, frequently are valid only for certain cases.

Presented article is devoted to non-stationary contact problem on wear of solid nonhomogeneous foundation of finite size in the shape of block under the action of moving rigid indentor. The wear speed was calculated on the basis of Archard model. There presented various results for wear amount for different Archard model parameters and two types of base nonhomogeneity.

There studied special cases for thin layer based on homogeneous foundation and case for linear function of elasticity modulus. Simulation was performed using finite element method and ANSYS software. Results are compared for particle case *and well known analytical solution for contact problem with wear for thin infinite layer. There demonstrated good satisfaction for both solutions.*

Keywords: *wear, friction, contact interaction, mathematical modelling.*

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