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DUALS FOR WEIGHTED SPACES OF HOLOMORPHIC FUNCTIONS OF PRESCRIBED GROWTH IN BOUNDED CONVEX DOMAINS

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We consider weighted (DFS)-spaces of holomorphic functions in a bounded convex domain that have a prescribed growth near its boundary. The growth assumptions are given by some general and natural conditions on the weights. Our main goal is to find a convenient description of the duals to this spaces in terms of the Laplace transform of functionals. In this direction the dual projective case of Fréchet spaces was mainly studied. Some meaningful results were obtained for only the space of holomorphic functions in a convex domain that have polynomial growth near its boundary. As an illustration of the application of this result an algorithm of constructing of a countable sufficient subset for the dual space is presented. The main result is also applied to the spaces of functions of exponential-power growth and to the construction of absolutely representing systems of exponentials in them. It was made a conclusion that there is an opportunity to represent of all functions from the main weight space by Dirichlet series that converge absolutely in this space.

Keywords: *weighted spaces of holomorphic functions, dual space, Laplace transform, absolutely representing systems, sufficient sets, Dirichlet series.*

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CALCULATION OF BOUNDARY CROSSING PROBABILITY FOR RANDOM PROCESSES WITH STABLE EVOLUTION OF ACTIVE SYSTEMS AND CALCULATION THE BARRIER OPTION PRICES IN SUBORDINATED MODELS*

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The purpose of this work is to find the fair price of a barrier option under the subordinated Balck - Sholes model with infinite activity subordinator, by approximating small jumps of subordinator and reducing the computation of barrier crossing probability to solving square inequality, by using stable property of Brownian motion and computing the price of an option with Monte Carlo technique.

Variance gamma process was considered as a subordinate with an infinite intensity of jumps.

Numerical experiments to calculate the fair price of an barrier option in subordinated model were conducted.

Results comparison of this method and the classical Monte Carlo method is given.

The advantage in the computational speed of the method over the classical Monte Carlo algorithm is noted.

Based on the study of the Wiener process subordination it is established that to find the probability of the barrier crossing, there is no need to completely reproduce the whole trajectory of the underlying asset, it is sufficient to simulate only the jump sizes and one normal random variable.

The problem of the relationship between the normal approximation error and intensity of jumps affecting computing costs was identified.

It is noted that the proposed approach, which relies on the stability property of the Wiener process, can be extended to arbitrary processes with the stability property.

It is concluded that the obtained results can be used not only in financial mathematics, but also in solving various problems related to the content of a random process in an admissible region, which is very important in solving problems related to the management of active systems.

Keywords: *Levy process, option pricing, Monte Carlo method, subordinator, model with jumps, Black - Sholes model.*

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FORECASTING OF EXPECTED VALUES OF FINANCIAL INDEXES

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The forecast problem and its general solution scheme in the terminology of the Hilbert space are considered in the paper. The linear prediction problem is posed as the design problem on the subspace of a Hilbert space. As an example, the Cox-Ross-Rubinshtein model with white noise is given, for which the algorithm for predicting the logarithmic return is obtained and the results of its software implementation are given. In the logarithmic return, there are two independent sources of randomness. As a single source of randomness, we consider a sequence of independent standard Gaussian random variables. For the second source of chance, two cases are given. In the first case, a sequence of independent and identically distributed binary random variables is used. In the second case, we use a sequence of independent random variables that form a Markov chain with given transition probabilities. For both cases, the dispersion of the error is calculated. The forecast of volatility in the model of stochastic volatility with the Kalman-Buci linear filter is also calculated, which is satisfactory and allows us to find the interval price forecast. It is shown that the filter is determined through a linear stochastic system of equations. The graphs of simulated and predicted values of stochastic volatility are given.

Keywords: *time series, linear forecast, stochastic volatility, Kalman-Buci linear filter, logarithmic return, white noise, Monte Carlo method, Markov chain, singular decomposition, Cox-Ross-Rubinstein model.*

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EFFECTIVELY COMPUTED VALUE FOR COOPERATIVE TU-GAME

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The subject of paper's study are the cooperative games in which the utility, achievable by a coalition, can be equally divided among the partners. The purpose of work is an investigation of properties of one-point solution that is determined for sufficiently wide class of games and have the polynomial computational complexity. That is a compromise-type solution. We give its axiomatic characterization using two standart axioms (efficiency and additivity) and two new axioms (modified null player property end restricted symmetry). We provide necessary and sufficient condition under which it belongs to nonempty imputation set as well as the sufficient conditions for its coincidence with the Shapley value and the τ-value. The field of applications for defined solution are the situations allowing subsidies. Reasonability of such solutions is motivated by necessity to take into account social aspects of cooperative game.

Keywords: *cooperative game, transferable utility, one-point solution, axiomatization, compromise.*

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CRITICAL STABILITY CASES OF EQUILIBRIA FOR TWO-COSYMMETRIC DIFFERENTIAL EQUATIONS

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The problem of equilibrium stability of two-cosymmetric differential equation is considered. We propose that these cosymmetries (vector fields are orthogonal to initial field in each point) satisfy conditions of the implicit function theorem for dynamical systems with cosymmetry. In particular, analyzed equilibrium is not cosymmetric, so it is not the equilibrium of neither of two cosymmetries. We consider the nondegeneracy condition: the linearization matrix has two-dimensional kernel. Thus the equilibrium is nonisolated and lies in two-dimensional continuous family of equilibria. Equilibrium stability is interpreted as neutral stability along the equilibria family and, moreover, asymptotic stability in transversal direction to the family. The critical case of the problem takes place than the stability spectrum does not contained an equilibrium with positive imaginary part and the neutral part σ *of the spectrum is not equal to double zero eigenvalue. The general case consists of six subcases, according to different stability spectra:* $\sigma_1 = \{0^2, 0\}$, $\sigma_2 = \{0, 0, \pm i\omega\}$, $\sigma_3 = \{0^3, 0\}$, $\sigma_4 = \{0^2, 0^2\}$, $\sigma_5 = \{0^2, 0, \pm i\omega\}$, $\sigma_6 = \{0, 0, \pm i\omega_1, \pm i\omega_2\}$. *For each subcase we constructed nonlinear model system and analyzed stability of its eqilibrium. The results are generalized to original system. In the subcases* $\sigma_1 - \sigma_5$ *instability is proved by Shnol method, which is related to the term "growing solution of the type of invariant ray", and in the subcase* σ_6 *we constructed corresponding Chetaev function. To prove stability we applied Lyapunov and Rumyatsev theorems about asymptotic stability with respect to some of the variables. The results for spectra* σ_1 *and* σ_2 *we generalized to the case of any finite codimension.*

Keywords: *cosymmetry, equilibrium, stability, direct Lyapunov method, codimension of degeneracy, invariant ray, dynamical system.*

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CAVITATION DECELERATION OF AN ELLIPTICAL CYLINDER IN A LIQUID AFTER IMPACT

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The two-dimensional problem about vertical continuous impact and the subsequent cavitational braking of the elliptic cylinder semi-shipped in a layer of an ideal incompressible fluid of finite depth is considered. As a result of a cavitational separation near the surface of a body the cavity is formed and the new internal free border of liquid appears. In general, the separation zone is a non-connected set. The solution of the problem is constructed by means of a direct asymptotic method, effective at small times. The special problem with unilateral restrictions on the basis of which the connectivity of a zone of a separation and also forms of external and internal free borders of liquid is defined is formulated. Because of the unknown zone of separation, this problem is non-linear and belongs to the class of problems with free boundaries. For a numerical solution of a problem with unilateral restrictions, a special iterative method is used, in which the previously unknown zones of detachment and contact of the liquid particles are subsequently refined. This non-linear problem reduces to a sequential solution of linear boundary value problems with fixed separation points. The latter problems are solved numerically, using the finite element method. The influence of the physical and geometric parameters of the problem on the main characteristics of the process is investigated. Specific examples are given showing the formation of caverns near the boundary of the body. The results obtained can be used to solve the practical problems of ship hydrodynamics.

Keywords: *ideal incompressible liquid, elliptical cylinder, impact, cavitational braking, asymptotics, free border, cavity, small times, Froude's number.*

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CONTACT PROBLEMS FOR AN INHOMOGENEOUS LAYER

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Three-dimensional contact problems of the elasticity theory are investigated on the interaction between rigid punches and an inhomogeneous layer with variable Poisson's ratio in depth and constant shear modulus. In this case the longitudinal modulus of elasticity is also variable in depth. The other layer face lies without friction on a rigid or Winkler base (problems A and B, respectively). The problems are reduced to an integral equation with respect to the contact pressures, the kernels are constructed in exact forms. For a priori unknown contact domain, the method of nonlinear boundary integral equations is used which make it possible to determine the contact zone and the contact pressures simultaneously. Calculations have been made for pyramidal and conical punches for different values of relative layer thickness as well as for different parameters of the nonhomogeneity law. Earlier for similar problems for more complicated functionally graded materials, approximations have been used for symbol functions of integral equations. In this paper, the nonhomogeneity law can serve as the simplest model which helps to discover the principal effects of the contact interaction.

Keywords: *contact problem, inhomogeneous elastic layer.*

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APPLICATION OF THE GLOBAL RANDOM SEARCH METHOD FOR CALCULATING THE FLOW OF A VISCOUS INCOMPRESSIBLE FLUID IN A CHANNEL

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In this paper, the plane problem of viscous incompressible fluid in a channel based on the Navier-Stokes equations and the continuity equation is considered. The stream function in this case can be considered for both laminar and turbulent modes. The main purpose is to test the mentioned algorithm on the example of the laminar flow. It is known that vortex structures with wave numbers determined by the Reynolds number play a dominant role in the turbulent flow. It is investigated the method based on the sufficiently overall approach of the residual functional minimization for operator equations. The residual functional is investigated in a quadratic metric. It is described by the global random search algorithm to minimize the residual functional. This algorithm is implemented by generating the sequence of random variables with consequent reduction of search ranges for the next iterations. It is assumed that the global random search algorithm provides high efficiency and significantly outperforms other numerical methods. Some examples of test functions for testing the global random search algorithm were demonstrated. It is proved that this method is well suited to the problem of residual functional minimization for the laminar flow.

Keywords: *Navier-Stokes equations, Reynolds number, Hagen-Poiseuille flow, stream function, laminar flow, random search, Rastrigin function.*

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BOUNDARY PROBLEM OF THE MEMBRANE THEORY CONVENTIONAL SHELLS FOR ONE CLASS OF SYMMETRIC DOME

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The some results related to the membrane theory of protuberant shells with the piece-smooth border of the it's middle surface is got. Within the framework of this theory the task of the realization of the momentless tense state of equilibrium of thin resilient shell is studied. The middle surface of S is interior of the ovaloid strictly positive gaussian curvature with the class of the regularity of $W^{3,p}$, $p \succ 2$, *and a the piece-smooth edge consisting of the finite number of arcs with the class of the regularity* $C^{1,\varepsilon}$, $0 < \varepsilon < 1$. *The development of this theory with generalized analytical functions approach requires the extended raising of the basic border task. Such raising is given for a shell with an onecoherent middle surface with the Riemann-Gilbert special border condition. The sufficient conditions of the quasicorrectnesses of the basic border task in the geometrical form are proved for the classes of surfaces (symmetric domes).*

Keywords: *convex shell, Riemann-Hilbert boundary value problem.*

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